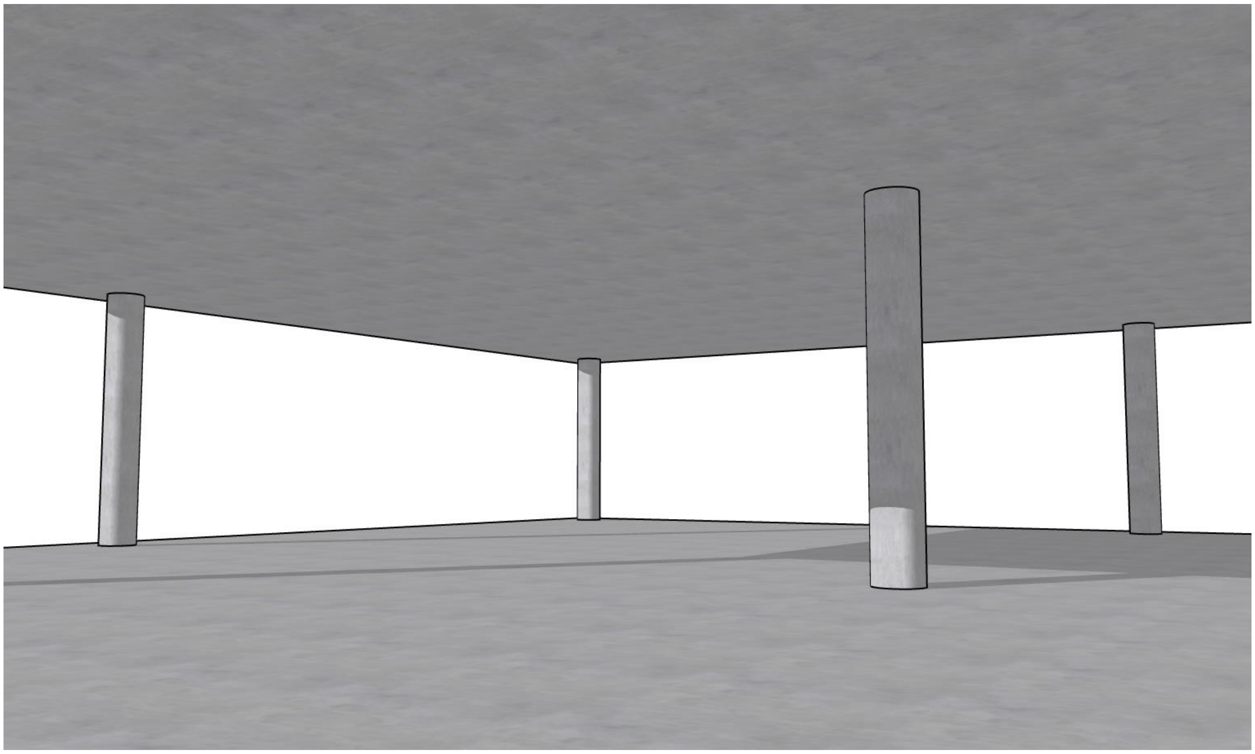
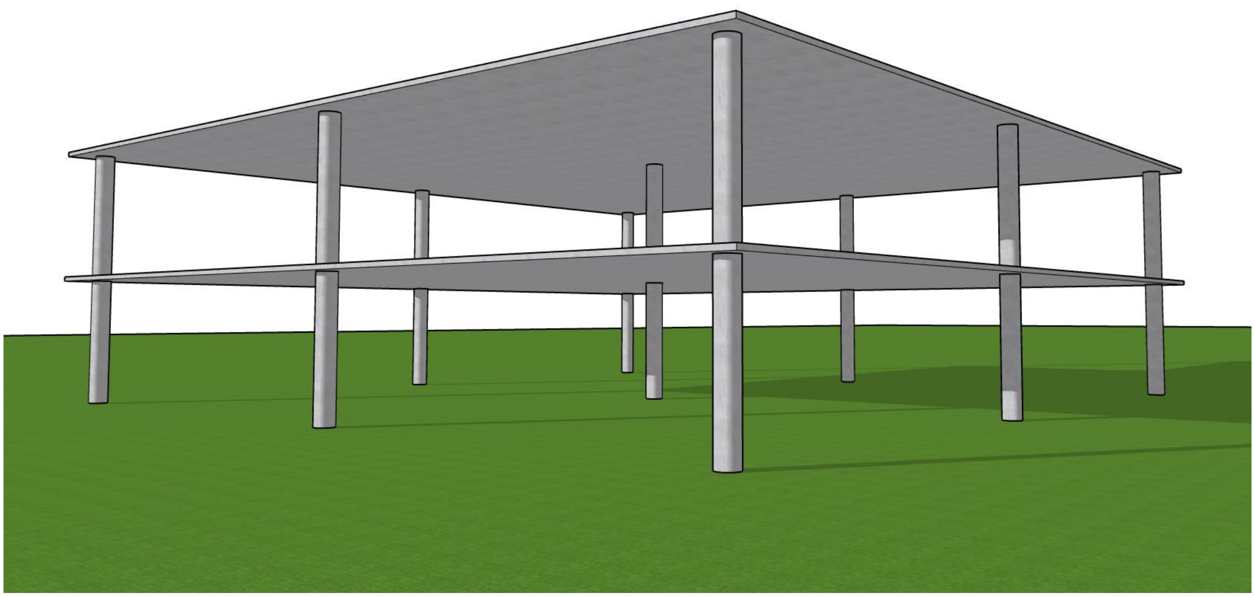
Two-way (Punching) Shear Calculations for Concrete Slab Supported by Circular Columns





Objective

Perform the two-way (punching) shear calculations around the exterior and interior circular columns supporting a two-way flat plate concrete slab. These calculations are widely published in text books for square and rectangular shapes but rarely are discussed in detail for circular columns or column capitals. This design example provides step-by-step hand-calculations and compares various ACI methodologies to determine the critical shear perimeter of circular columns.

Codes

Building Code Requirements for Structural Concrete (ACI 318-95) and Commentary (ACI 318R-95)

Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05)

Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary (ACI 318R-11)

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

[1] Vanderbilt, M.D., "Shear Strength of Continuous Plates," Journal of Structural Division, ASCE, V. 98, No. ST5, May 1972, pp. 961-973.

[2] ACI-ASCE Committee 426, "The Shear Strength of Reinforced Concrete Members," Chapter 5, "Shear Strength of Slabs," Proceedings ASCE, Journal of the Structural Division, Vol. 100, No. ST8, August 1974, pp. 1543–1591.

[3] ACI-ASCE Committee 426, "Suggested Revisions to Shear Provisions for Building Codes," Proc. ACI, Vol. 74, September 1977, pp. 458-469.

[4] ACI Design Handbook, ACI 340R-97 [SP-17 (97)], Shear Example 9

[5] MacGregor J.G., Bartlett F.M., Reinforced Concrete – Mechanics and Design, First Canadian Edition, Prentice Hall Canada Inc., 2000

[6] ACI Design Handbook, SP-17 (09) in accordance with ACI 318-05

[7] ACI Design Handbook, SP-17 (11) in accordance with ACI 318-11

[8] ACI-352.1R-11 Guide for Design of Slab-Column Connections in Monolithic Concrete Structure

[9] Notes on ACI 318-11 Building Code Requirements for Structural Concrete with Design Applications, Edited by Mahmoud E. Kamara and Lawrence C. Novak, Portland Cement Association, 2013

[10] Wight J.K., Reinforced Concrete, Mechanics and Design, Seventh Edition, Pearson Education, Inc., 2016

Analysis and Design Data

f_c' = 5,000 psi and f_y = 40,000 psi

l_1 = 20'-0" and l_2 = 18'-0"

Uniformly distributed factored load, w_u = 986 psf (includes self-weight of slab, live load, and superimposed dead load)

Supporting circular column diameter, h_c = 34 in.

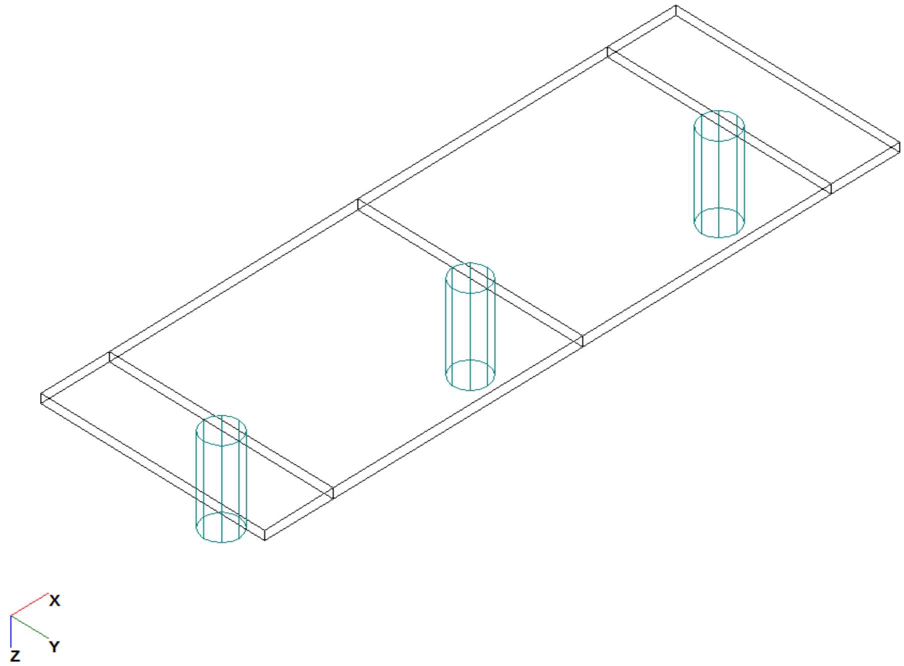
Effective depth for two-way shear, d = 10.3 in. (average of effective depths in two orthogonal direction per ACI 318)

Strength reduction factor, ϕ, for shear equals to 0.75 per ACI 318-14

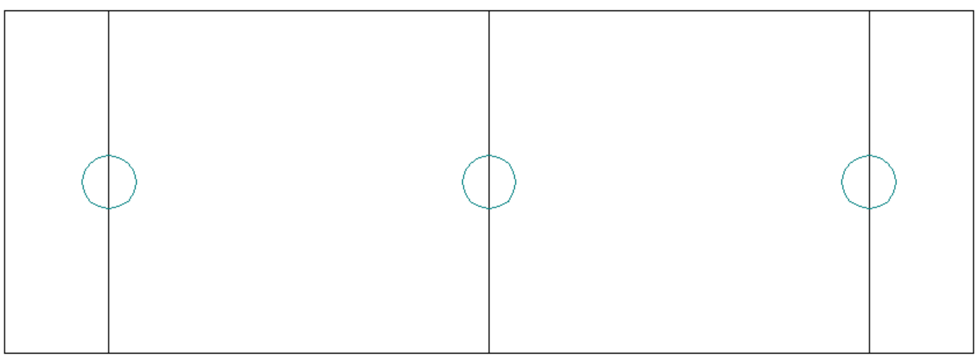
Contents

Two-way Slab Model

The isometric and plan views below are produced by the spSlab Program.



Isometric view of two-way flat plate concrete floor slab supported by circular columns



Exterior Column
34 in. diameter

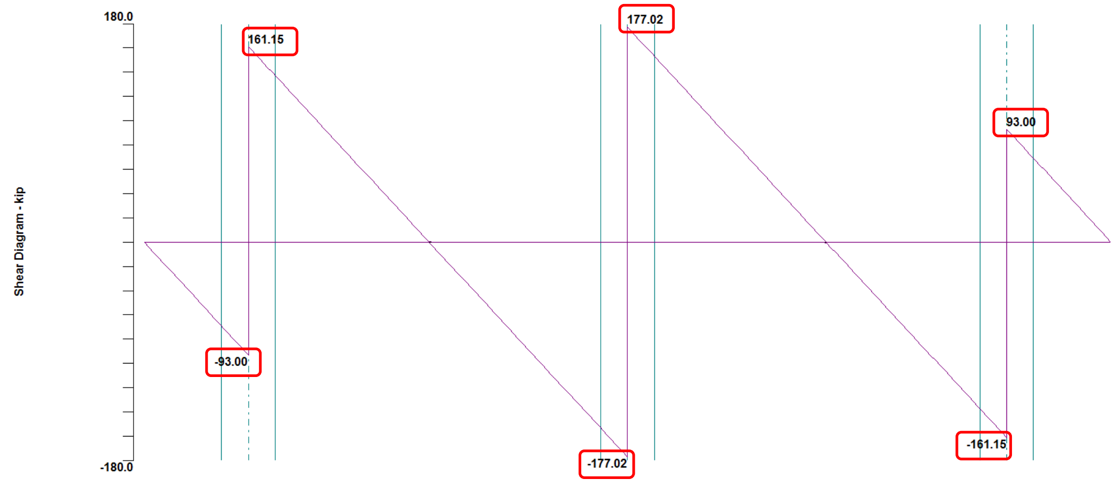
Interior Column
34 in. diameter

Exterior Column
34 in. diameter

Plan view of two-way flat plate concrete floor slab supported by circular columns

Internal Force (Shear Force & Bending Moment) Diagrams

The shear force and bending moment diagrams below are produced by the spSlab Program and will be used for two-way (punching) shear calculations.

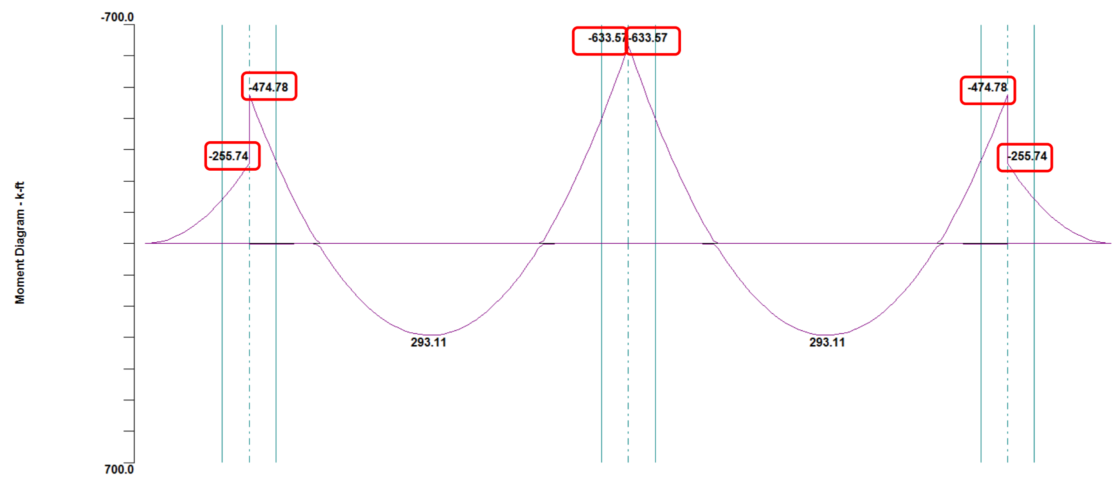


Shear Force Diagram from spSlab

From the shear force diagram, the reactions at the centroid of the critical section for exterior and interior columns are:

At exterior supporting column, the reaction, R = [93.00 + 161.15] = 254.15 kips

At interior supporting column, the reaction, R = [177.02 + 177.02] = 354.04 kips



Bending Moment Diagram from spSlab

From the bending moment diagram, the factored unbalanced moment used for shear transfer, M_{unb}, for exterior and interior columns are:

At exterior supporting column, the unbalanced moment, M_{unb} = [474.78 – 255.74] = 219.04 k-ft

At interior supporting column, the unbalanced moment, M_{unb} = [633.57 – 633.57] = 0.00 k-ft

Two-way (punching) Shear Calculations

Two-way (punching) shear calculations are performed to ensure that the concrete slab design shear strength, ϕv_n, shall be greater than or equal to the factored shear stress, v_u.

$$\phi v_n \geq v_u$$

The combined two-way (punching) shear stress, v_u, is calculated as the summation of direct shear alone and direct shear transfer resulting from the unbalanced moment:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_{unb} c}{J}$$

The factored shear force, V_u, at the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section perimeter.

The factored unbalanced moment used for shear transfer, M_{unb}, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

Without shear reinforcement in the slab, the equivalent concrete stress corresponding to nominal two-way shear strength of slab, v_n, equals to the stress corresponding to nominal two-way shear strength provided by concrete, v_c.

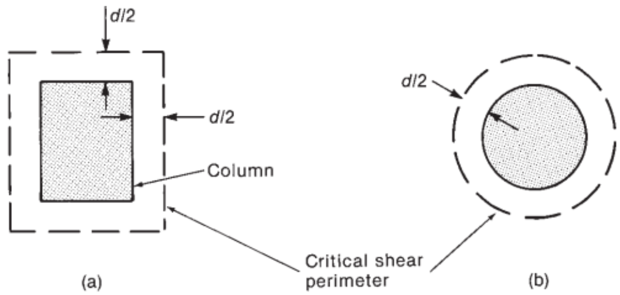
$$v_n = v_c = \min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\frac{\alpha_s d}{b_0}+2\right)\lambda\sqrt{f_c'}\right] \quad \text{per ACI 318-95 thru ACI 318-19}$$

β = the ratio of the long to the short side of the supporting column (equals to 1.0 for circular columns)

α_s = a constant dependent on supporting column location (equals to 40 for an interior effective critical area)

λ = 1.0 (normal weight concrete)

b_0 = the perimeter of the critical section for two-way shear. The critical section shall be located so that the perimeter, b_0, is a minimum but need not be closer than d/2 to the perimeter of the supporting column per ACI 318-14, 22.6.4.1.

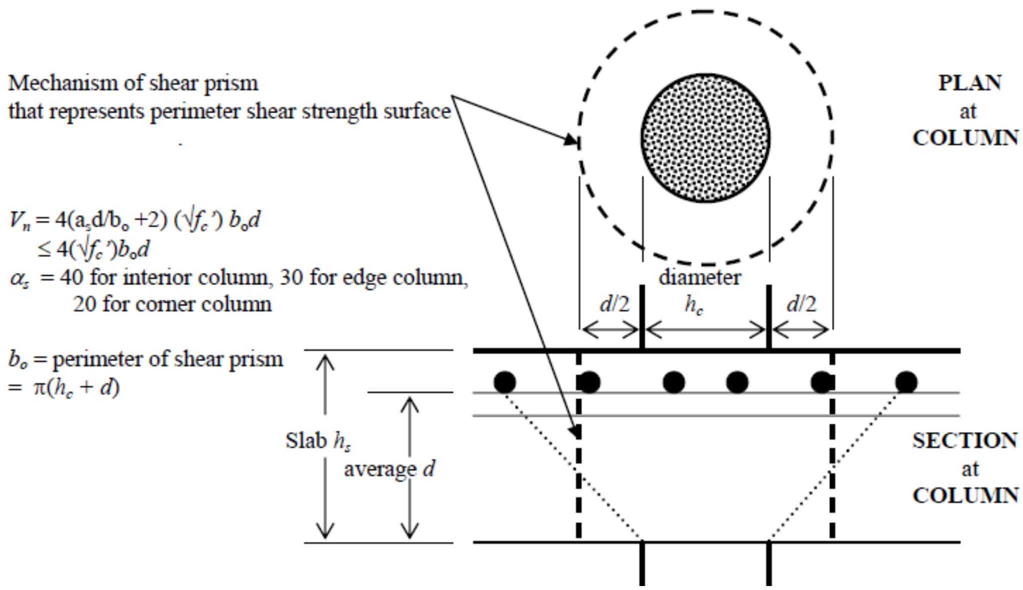


Location of critical shear perimeter – Excerpt from pg. 689 of Reference [10]

The two-way shear calculations based on an exact circular critical shear perimeter of circular interior and exterior supporting column as shown above along with two approximate critical shear perimeter methods will be discussed.

Method 1: Exact Circular Critical Shear Perimeter - ACI Design Handbook [SP-17][4] [6] [7] & ACI 318-11

The ACI Design Handbook [from SP-17 (97) thru SP-17 (11)] utilizes exact circular critical shear perimeter located d/2 away from circular column perimeter as per the *"Provisions for slabs and footings"* within chapter 11 of ACI 318 [ACI 318-95, 11.12.1.2 thru ACI 318-11, 11.11.1.2, and ACI 318-14, 22.6.4.1] for two-way (punching) shear calculations.



Excerpt from Shear Example 9 of SP-17(9)[6]: ACI Design Handbook in accordance with ACI 318-05

Circular critical section properties for two-way shear stress calculations for circular column are shown below.

Critical section for circular column, b_0 is based on circular perimeter.

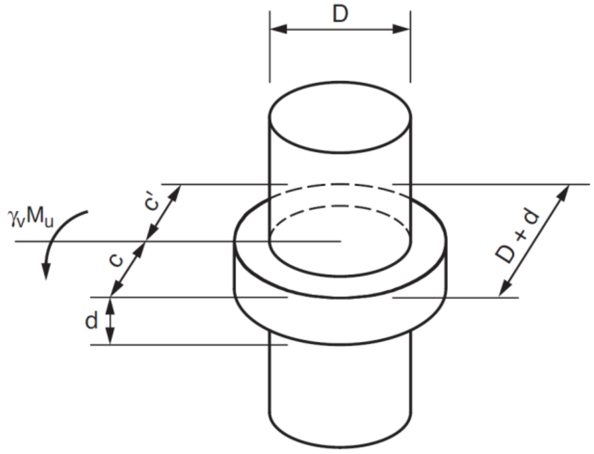
$b_0 = \pi(h_c + d)$

Area of circular concrete section resisting shear transfer, A_c, equals to circular perimeter of critical section, b_0, multiplied by the effective depth, d.

$A_c = b_0 \times d = \pi(h_c + d) \times d$

$c = c' = \frac{(h_c + d)}{2}$

$\frac{J}{c} = \pi d \times \left[\frac{(h_c + d)}{2}\right]^2 + \frac{d^3}{3}$



Excerpt from Fig. 16-14 of PCA Notes on ACI 318-11[9]
where D equals to the diameter of circular column, h_c

For the combined two-way (punching) shear stress, v_u, calculations, circular critical section properties of exterior and interior columns are identical due to the existence of 6-ft cantilever span at exterior columns.

$$b_0 = \pi \times (h_c + d) = \pi \times (34 + 10.3) = 139.18 \text{ in.}$$

$$A_c = b_0 \times d = \pi \times (h_c + d) \times d = \pi \times (34 + 10.3) \times 10.3 = 1433.5 \text{ in}^2$$

$$c = c' = \frac{(h_c + d)}{2} = \frac{34 + 10.3}{2} = 22.15 \text{ in.}$$

$$\frac{J}{c} = \pi \times d \times \left[\frac{(h_c + d)}{2}\right]^2 + \frac{d^3}{3} = \pi \times 10.3 \times \left[\frac{(34 + 10.3)}{2}\right]^2 + \frac{10.3^3}{3} = 16240 \text{ in}^3$$

Sum of the self-weight and superimposed surface dead and live load acting within the critical section perimeter is:

$$w_u\left(\frac{\pi \times (h_c + d)^2}{4}\right) = \frac{0.986}{144}\left(\frac{\pi \times (34 + 10.3)^2}{4}\right) = 10.55 \text{ kips}$$

Exterior supporting column:

The two-way combined shear stress, v_u, can be calculated as:

$$v_u = \frac{V_u}{b_0 \times d} + \frac{\gamma_v M_{unb} c}{J} \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 254.15 kips from shear diagram and the unbalanced moment, M_{unb} = 219.04 k-ft from bending moment diagram.

$$v_u = \frac{(254.15 - 10.55) \times 1,000}{139.18 \times 10.3} + \frac{0.4 \times [(219.04) \times 12,000]}{16,240}$$

$$v_u = 169.9 + 64.7 = 234.6 \text{ psi}$$

The two-way design shear strength without shear reinforcement, ϕv_n, can be calculated as:

$$\phi v_n = \phi v_c = \phi \times \min\left[4\lambda\sqrt{f_c'} \,,\, \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f_c'} \,,\, \left(\frac{\alpha_s d}{b_0} + 2\right)\lambda\sqrt{f_c'}\right]$$

$$\phi v_n = \phi v_c = 0.75 \times \min[282.8 \,,\, 424.3 \,,\, 350.7] = 0.75 \times 282.8 = 212.1 \text{psi}$$

$$v_u / \phi v_n = 234.6 / 212.1 = 1.11 > 1.0 \text{ N.G.}$$

Since $\phi v_n < v_u$ at the critical section, the slab has **inadequate** two-way shear strength at the exterior column.

Interior supporting column:

The two-way combined shear stress, v_u, can be calculated as:

$$v_u = \frac{V_u}{b_0 \times d} + \frac{\gamma_v M_{unb} c}{J} \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 354.04 kips from shear diagram and the unbalanced moment, M_{unb} = 0 k-ft from bending moment diagram.

$$v_u = \frac{(354.04 - 10.55) \times 1,000}{139.18 \times 10.3} + \frac{0.4 \times [(0) \times 12,000]}{16,240}$$

$$v_u = 239.6 + 0.0 = 239.6 \text{ psi}$$

The two-way design shear strength without shear reinforcement, ϕv_n, can be calculated as:

$$\phi v_n = \phi v_c = \phi \times \min\left[4\lambda\sqrt{f_c'} \,,\, \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f_c'} \,,\, \left(\frac{\alpha_s d}{b_0} + 2\right)\lambda\sqrt{f_c'}\right]$$

$$\phi v_n = \phi v_c = 0.75 \times \min[282.8 \,,\, 424.3 \,,\, 350.7] = 0.75 \times 282.8 = 212.1 \text{psi}$$

$$v_u / \phi v_n = 239.6 / 212.1 = 1.13 > 1.0 \text{ N.G.}$$

Since $\phi v_n < v_u$ at the critical section, the slab has **inadequate** two-way shear strength at the interior column.

Method 2: Approximate Square Critical Shear Perimeter based on an Equivalent Square Column Perimeter that is equal to Circular Supporting Column Perimeter- ACI 352.1R-11 [8] & References [1], [2], [3], [5]

ACI 352.1R-11 states that "Punching shear strengths for connections with circular columns have been observed (Vanderbilt 1972[1]), to exceed the punching shear strength for connections with square columns having the same perimeter (periphery) (Fig 3.1b(c))." The ACI-ASCE Committee 426 [3] indicates the same by stating that the critical shear perimeter, b_0 value of $(\pi h_c + 4d)$ permitted for circular loaded areas (Method 2) exceeds the $\pi(h_c + d)$ value for a critical section d/2 from the perimeter of the circular loaded area (Method 1). It continues by stating that liberalization (use of larger b_0 value (Method 2) as compared to exact b_0 value (Method 1) is in accordance with the findings of Committee 426[2]. And the significant finding No. 4 of Reference [2] stats that "strength for loading through circular areas are greater than for loading through the square areas. More realistic capacities for circular areas can be restricted by using a square critical section with side length of $[\pi/4)c + d]$.(as shown in figures below)

The perimeter of critical section for circular column, b_0, is based on a square column with the same centroid and the same length of perimeter.

$$b_0 = 4\times\left(\frac{\pi}{4}h_c + d\right) = (\pi h_c + 4d)$$

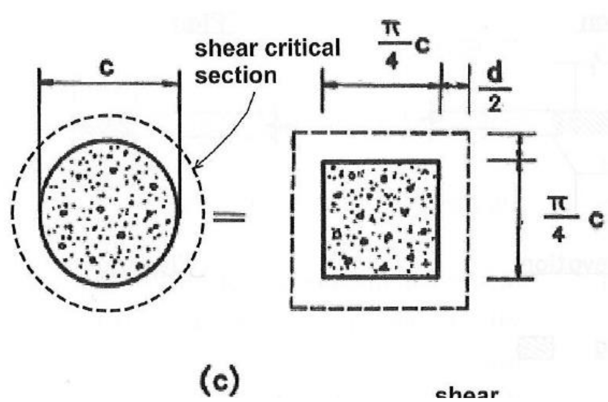
Area of square concrete section resisting shear transfer, A_c, equals to square perimeter of critical section, b_0, multiplied by the effective depth, d.

$$A_c = b_0 \times d = 4\times\left(\frac{\pi}{4}h_c + d\right)\times d$$

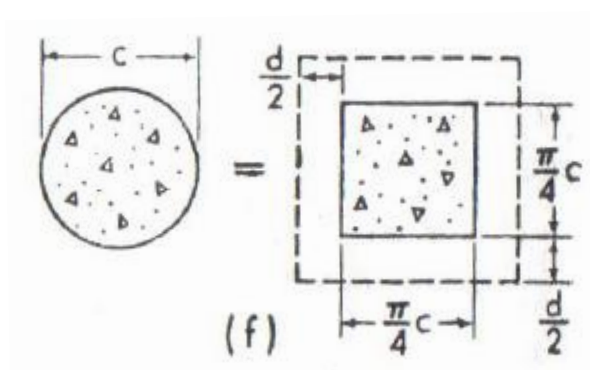
$$c = c' = \frac{\left(\frac{\pi}{4}h_c + d\right)}{2}$$

$$\frac{J}{c} = \frac{4\times\left[\left(\frac{\pi}{4}h_c + d\right)^2 \times d\right] + d^3}{3}$$

In Method 2, the side dimension for the equivalent square column would be $\frac{\pi}{4}h_c$ (i.e. $0.785h_c$ as it appears at Fig. 13-57 of Reference [5])



Excerpt from Fig. 3.1b of ACI 352.1R-11[8] where c equals to the diameter of circular column, h_c.



Excerpt from Fig. 7 of ACI-ASCE Committee 426 [3] where c equals to the diameter of circular column, h_c.

From the equations above:

$$b_0 = 4\times\left(\frac{\pi}{4}h_c + d\right) = 4\times\left(\frac{\pi}{4}\times 34 + 10.3\right) = 148.01 \text{ in.}$$

$$A_c = b_0 \times d = 4\times\left(\frac{\pi}{4}h_c + d\right)\times d = 4\times\left(\frac{\pi}{4}\times 34 + 10.3\right)\times 10.3 = 1524.5 \text{ in}^2$$

$$c = c' = \frac{\left(\frac{\pi}{4}h_c + d\right)}{2} = \frac{\left(\frac{\pi}{4}\times 34 + 10.3\right)}{2} = 18.50 \text{ in.}$$

$$\frac{J}{c} = \frac{4\times\left[\left(\frac{\pi}{4}h_c + d\right)^2 \times d\right] + d^3}{3} = \frac{4\times\left[\left(\frac{\pi}{4}\times 34 + 10.3\right)^2 \times 10.3\right] + (10.3)^3}{3} = 19{,}168.8 \text{ in}^3$$

Sum of the self-weight and superimposed surface dead and live load acting within the critical section perimeter is:

$$w_u\left(\frac{\pi}{4}h_c + d\right)^2 = \frac{0.986}{144}\left(\frac{\pi}{4}\times 34 + 10.3\right)^2 = 9.38 \text{ kips}$$

Exterior supporting column:

The two-way combined shear stress, v_u, can be calculated as:

$$v_u = \frac{V_u}{b_0 \times d} + \frac{\gamma_v M_{unb} c}{J} \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 254.15 kips from shear diagram and the unbalanced moment, M_{unb} = 219.04 k-ft from bending moment diagram.

$$v_u = \frac{(254.15 - 9.38)\times 1{,}000}{148.01\times 10.3} + \frac{0.4\times[(219.04)\times 12{,}000]}{19{,}168.8}$$

$$v_u = 160.6 + 54.8 = 215.4 \text{ psi}$$

For this exterior column, the liberalization attributed to this method in Reference [3] leads to 5.5% lower direct shear stress [160.6 psi (Method 2) vs. 169.9 psi (Method 1)], 15.3% lower shear stress due to unbalanced moment [54.8 psi (Method 2) vs. 64.7 psi (Method 1)]. Therefore, the two-way combined shear stress, v_u, value of 215.4 psi from Method 2 is 8.2% lesser in magnitude and still acceptable (the liberalization) as compared to the value of 234.6 psi from the exact circular critical shear perimeter method (Method 1).

The two-way design shear strength without shear reinforcement, ϕv_n, can be calculated as:

$$\phi v_n = \phi v_c = \phi \times \min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\frac{\alpha_s d}{b_0}+2\right)\lambda\sqrt{f_c'}\right]$$

$$\phi v_n = \phi v_c = 0.75 \times \min[282.8\ ,\ 424.3\ ,\ 338.2] = 0.75 \times 282.8 = 212.1\ \text{psi}$$

$$v_u / \phi v_n = 215.4 / 212.1 = 1.02 > 1.0\ \text{N.G.}$$

Since $\phi v_n < v_u$ at the critical section, the slab has **inadequate** two-way shear strength at the exterior column.

Interior supporting column:

The two-way combined shear stress, v_u, can be calculated as:

$$v_u = \frac{V_u}{b_0 \times d} + \frac{\gamma_v M_{unb} c}{J} \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 354.04 kips from shear diagram and the unbalanced moment, M_{unb} = 0 k-ft from bending moment diagram.

$$v_u = \frac{(354.04 - 9.38) \times 1{,}000}{148.01 \times 10.3} + \frac{0.4 \times [(0) \times 12{,}000]}{19{,}168.8}$$

$$v_u = 226.1 + 0.0 = 226.1\ \text{psi}$$

For this interior column with zero unbalanced moment, the liberalization attributed to this method in Reference [3] leads to 5.6% lower direct shear stress [226.1 psi (Method 2) vs. 239.6 psi (Method 1)]. Therefore, the two-way combined shear stress, v_u, value of 226.1 psi from Method 2 is 5.6% lesser in magnitude and still acceptable (the liberalization) as compared to the value of 239.6 psi from the exact circular perimeter method (Method 1).

The two-way design shear strength without shear reinforcement, ϕv_n, can be calculated as:

$$\phi v_n = \phi v_c = \phi \times \min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\frac{\alpha_s d}{b_0}+2\right)\lambda\sqrt{f_c'}\right]$$

$$\phi v_n = \phi v_c = 0.75 \times \min[282.8\ ,\ 424.3\ ,\ 338.2] = 0.75 \times 282.8 = 212.1\ \text{psi}$$

$$v_u / \phi v_n = 226.1 / 212.1 = 1.07 > 1.0\ \text{N.G.}$$

Since $\phi v_n < v_u$ at the critical section, the slab has **inadequate** two-way shear strength at the interior column.

Method 3: Approximate Square Critical Shear Perimeter based on an Equivalent Square Column Area that is equal to Circular Supporting Column Area - ACI 318-14, 22.6.4.1.2

ACI 318-14 introduced new provision regarding the critical sections of circular columns for two-way shear calculations. This new clause (ACI 318-14, 22.6.4.1.2) states that "for a circular or regular polygon-shaped column, critical sections for two-way shear in accordance with 22.6.4.1(a) and (b) shall be permitted to be defined assuming a square column of equivalent area.

The perimeter of critical section for circular column, b_0, is based on a square column with the same centroid and the same area.

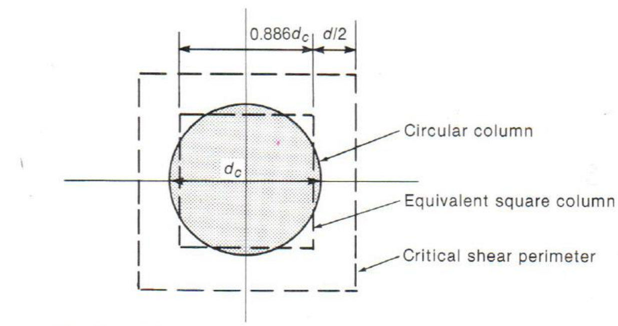
$$b_0 = 4\times\left(\sqrt{\frac{\pi}{4}}h_c + d\right)$$

Area of square concrete section resisting shear transfer, A_c, equals to square perimeter of critical section, b_0, multiplied by the effective depth, d.

$$A_c = b_0 \times d = 4\times\left(\sqrt{\frac{\pi}{4}}h_c + d\right)\times d$$

$$c = c' = \frac{\left(\sqrt{\frac{\pi}{4}}h_c + d\right)}{2}$$

$$\frac{J}{c} = \frac{4\times\left[\left(\sqrt{\frac{\pi}{4}}h_c + d\right)^2 \times d\right] + d^3}{3}$$



(a) Interior column.

Excerpt from Fig. 13-77 of Wight J.K. Reinforced Concrete, Mechanics and Design, Seventh Edition [10] where d_c equals to the diameter of circular column, h_c.

In Method 3, the side dimension for the equivalent square column would be $\sqrt{\frac{\pi}{4}}h_c$ (i.e. $0.886h_c$)

From the equations above:

$$b_0 = 4\times\left(\sqrt{\frac{\pi}{4}}h_c + d\right) = 4\times\left(\sqrt{\frac{\pi}{4}}\times 34 + 10.3\right) = 161.7 \text{ in.}$$

$$A_c = b_0 \times d = 4\times\left(\sqrt{\frac{\pi}{4}}h_c + d\right)\times d = 4\times\left(\sqrt{\frac{\pi}{4}}\times 34 + 10.3\right)\times 10.3 = 1665.8 \text{ in}^2$$

$$c = c' = \frac{\left(\sqrt{\frac{\pi}{4}}h_c + d\right)}{2} = \frac{\left(\sqrt{\frac{\pi}{4}}\times 34 + 10.3\right)}{2} = 20.22 \text{ in.}$$

$$\frac{J}{c}=\frac{4\times\left[\left(\sqrt{\frac{\pi}{4}}h_c+d\right)^2\times d\right]+d^3}{3}=\frac{4\times\left[\left(\sqrt{\frac{\pi}{4}}\times 34+10.3\right)^2\times 10.3\right]+(10.3)^3}{3}=22{,}814.4\ \text{in}^3$$

Sum of the self-weight and superimposed surface dead and live load acting within the critical section perimeter is:

$$w_u\left(\sqrt{\frac{\pi}{4}}h_c+d\right)^2=\frac{0.986}{144}\left(\sqrt{\frac{\pi}{4}}\times 34+10.3\right)^2=11.19\ \text{kips}$$

Exterior supporting column:

The two-way combined shear stress, v_u , can be calculated as:

$$v_u=\frac{V_u}{b_0\times d}+\frac{\gamma_v M_{unb}c}{J}\quad \text{where } \gamma_v=0.40$$

The reaction, R = 254.15 kips from shear diagram and the unbalanced moment, M_{unb} = 219.04 k-ft from bending moment diagram.

$$v_u=\frac{(254.15-11.19)\times 1{,}000}{161.7\times 10.3}+\frac{0.4\times[(219.04)\times 12{,}000]}{22{,}814.4}$$

$$v_u=145.9+46.1=193.0\ \text{psi}$$

For this exterior column, this method per ACI 318-14 leads to 14.1% lower direct shear stress [145.9 psi (Method 3) vs. 169.9 psi (Method 1)], 28.7% lower shear stress due to unbalanced moment [46.1 psi (Method 3) vs. 64.7 psi (Method 1)]. Therefore, the two-way combined shear stress, v_u ,value of 193.0 psi from Method 3 is 17.7% lesser in magnitude as compared to the value of 234.6 psi from the exact circular critical shear perimeter method (Method 1).

The two-way design shear strength without shear reinforcement, ϕv_n , can be calculated as:

$$\phi v_n=\phi v_c=\phi\times\min\left[4\lambda\sqrt{f_c'}\ ,\ \left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}\ ,\ \left(\frac{\alpha_s d}{b_0}+2\right)\lambda\sqrt{f_c'}\right]$$

$$\phi v_n=\phi v_c=0.75\times\min[282.8\ ,\ 424.3\ ,\ 321.6]=0.75\times 282.8=212.1\ \text{psi}$$

$$v_u/\phi v_n=193.0/212.1=0.91<1.0\ \text{O.K.}$$

Since $\phi v_n \geq v_u$ at the critical section, the slab has **adequate** two-way (punching) shear strength at the exterior column. It is important to point out the slab at exterior column that was deemed inadequate for two-way shear strength per Methods 1 and 2, appears not only to be adequate but to have extra 9.0% reserve capacity per the Method 3.

Interior supporting column:

The two-way combined shear stress, v_u , can be calculated as:

$$v_u = \frac{V_u}{b_0 \times d} + \frac{\gamma_v M_{unb} c}{J} \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 354.04 kips from shear diagram and the unbalanced moment, M_{unb} = 0 k-ft from bending moment diagram.

$$v_u = \frac{(354.04 - 11.19) \times 1,000}{161.7 \times 10.3} + \frac{0.4 \times [(0.00) \times 12,000]}{22,814.4}$$

$$v_u = 205.9 + 0.0 = 205.9 \text{ psi}$$

For this interior column with zero unbalanced moment, this method per ACI 318-14 leads to 14.1% lower direct shear stress [205.9 psi (Method 3) vs. 239.6 psi (Method 1)]. Therefore, the two-way combined shear stress, v_u ,value of 205.9 psi from Method 3 is 14.1% lesser in magnitude as compared to the value of 239.6 psi from the exact circular critical shear perimeter method (Method 1).

The two-way design shear strength without shear reinforcement, ϕv_n , can be calculated as:

$$\phi v_n = \phi v_c = \phi \times \min\left[4\lambda\sqrt{f_c'} , \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f_c'} , \left(\frac{\alpha_s d}{b_0} + 2\right)\lambda\sqrt{f_c'}\right]$$

$$\phi v_n = \phi v_c = 0.75 \times \min[282.8 , 424.3 , 321.6] = 0.75 \times 282.8 = 212.1 \text{ psi}$$

$$v_u / \phi v_n = 205.9 / 212.1 = 0.97 < 1.0 \text{ O.K.}$$

Since $\phi v_n \geq v_u$ at the critical section, the slab has **adequate** two-way (punching) shear strength at the interior column. It is important to point out the slab at interior column that was deemed inadequate for two-way shear strength per Methods 1 and 2, appears not only to be adequate but to have extra 3.0% reserve capacity per the Method 3.

Summary of Results

The results related to two-way (punching) shear calculations per three methods are summarized in table below:

Method Used	**Method 1**	**Method 2**	Diff.	**Method 3**	Diff.
Critical Shear Perimeter	Exact Circular	Approximate Square		Approximate Square	
Column Geometry utilized for Critical Shear Perimeter Calculation	Circular Supporting Column	Square Column with Same Perimeter as the Circular Supporting Column		Square Column with Same Area as the Circular Supporting Column	
Side Length of Square Column where h_c is the diameter of circular column	N/A	$\frac{\pi}{4}h_c$ $(0.785h_c)$		$\sqrt{\frac{\pi}{4}}h_c$ $(0.886h_c)$	
Supporting Codes and References	ACI 318-11 and prior editions ACI 340R-97 [4] SP-17 (09) [6] SP-17 (11) [7]	ACI 352-1R-11 [8] ACI-ASCE Committee 426 [2] [3] Reference [1] Reference [5]		ACI-318-14 ACI 318-19 Reference [10]	
The perimeter of the critical section, b_0 (in)	139.2	148.0	+6.3%	161.7	+16.2%
Area of circular concrete section resisting shear transfer, A_c (in^2)	1,433.5	1,524.5	+6.3%	1,665.8	+16.2%
J/c (in^3)	16,240	19,169	+18.0%	22,814	+40.5%
Exterior Column					
The two-way combined shear stress, v_u (psi)	234.6	215.4	-8.2%	193.0	-17.7%
The two-way design shear strength without shear reinforcement, ϕv_n (psi)	212.1	212.1	0%	212.1	0%
Capacity Ratio $v_u / \phi v_n$ ($\leq$1.00 is Safe)	1.11	1.02		0.91	
Status	**Inadequate**	**Inadequate**		**Adequate**	
Interior Column					
The two-way combined shear stress, v_u (psi)	239.6	226.1	-5.6%	205.9	-14.1%
The two-way design shear strength without shear reinforcement, ϕv_n (psi)	212.1	212.1	0%	212.1	0%
Capacity Ratio $v_u / \phi v_n$ ($\leq$1.00 is Safe)	1.13	1.07		0.97	
Status	**Inadequate**	**Inadequate**		**Adequate**	